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The Three Dice Problem

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Abstract. We describe a five-step problem-solving exercise that is designed to introduce students to the concepts of probability estimates, revising probability estimates to incorporate additional information, and computing the expected value of an information signal in the context of a business problem. We occasionally expand discussion to include prospect theory and enumeration fallacies when time is available. We have used the exercise numerous times with graduate business students enrolled in core operations management, supply chain, and statistics courses. Students typically begin working as individuals and then break into groups as the problems become more complex. The problem never fails to generate excitement in the room and provides an excellent kickoff to an operations management class or degree program.

Keywords*:* expected value, value of information, model evaluation.

1. Introduction

Many of the techniques and much of the content taught in our operation management courses involve building a model to improve managerial decision making. For example, we teach techniques such as linear programming and economic order quantity (EOQ) to determine how much inventory should be produced or ordered to minimize costs. We teach queueing theory to determine how many employees or service stations should be available in order to effectively manage anticipated demand. And additional models are used for project planning, forecasting, and statistical process control, all of which identify, collect, and analyze information that can be brought to bear to improve managerial decision making. The underlying assumption when implementing these techniques is that the cost of carrying out these methods is less than the benefits one hopes to gain from using them. This type of cost-benefit analysis is sometimes missing from textbooks. In our experience, however, most faculty explicitly incorporate this concept into their lectures or, in our case, positioning of the core operations management concepts. We present in the paper a version of "The Three Dice Problem" that we have been using with graduate and executive students for years (five decades for one colleague in particular).

There are two primary learning objectives, as well as some general principles, that we seek to achieve by using this problem. The first objective

is to explain how decision-making alternatives can be evaluated by examining the incremental changes in the probability of making a good decision. Multiple decision-making strategies can be compared, in part, by the likelihood that using them will lead to better decisions. The second objective is to look at the value associated with good decision making and extend this to the concepts of *expected value* and the associated concept of the *value of information*. These ideas are familiar to anyone who teaches operations management or decision sciences but are nonetheless essential to both. We find this exercise to be a good one to use on the first day of class or early in the semester. The exercise can serve as an effective way to position model building as well as anchor a more general discussion about preparing oneself to succeed as a professional in the workplace. We have added some secondary objectives over the years, as we demonstrate below, the learning activity can be used to introduce or reinforce the concepts of prospect theory, enumeration fallacies and the central limit theorem.

We proceed by presenting the setup of the problem, followed by a series of five questions, each building on the former, along with some ideas for positioning this in the classroom. We invite the reader to work through the problem as it is presented to our students, without first knowing the answers. To facilitate this in the present paper we have presented the setup with two figures (Figures 1 & 2) that we use in the classroom. These are provided by a series of five questions presented throughout the paper. The reader may wish to review the Figures and work through the questions and then return to the text to read the discussion from the instructor's perspective.

2. The Setup

The problem is based on three different and well-balanced six-sided dice presented at random to the participant, and a series of probabilistic questions about their identity. The names and numbers of the three dice are as follows:

Fair Die: {1, 2, 3, 4, 5, 6}
Even Die: {2, 2, 4, 4, 6, 6}
Sixes Die: {6, 6, 6, 6, 6, 6}

Care must be taken to ensure that the students understand this setup. We've found that a picture of the three die (Figure 1) can effectively communicate the arrangement.

The first question begins by explaining to the students that the dice are presented to them in random order. This is done with each die having a six face up on it. It is helpful here if the instructor walks to the center of the room, looks down at the floor, and tells the students to imagine they are looking down at the three dice. Extraordinary care should be taken to explain this relatively