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The Monty Hall Problem: Use of an Interactive Excel Application to Encourage Critical Thinking and Illustrate Probability Concepts in Operations and Supply Chain Management Courses

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Abstract. This teaching brief describes an active learning workshop to help students understand basic ideas regarding conditional probability, apply Bayes' Theorem to operational problems, and see for themselves that sometimes one's intuition may be fallible. We use the interesting, and often perplexing Monty Hall problem to introduce students to probability concepts in an engaging critical thinking exercise. Students engage in an Excel simulation so they can see for themselves implications of decisions made. Instructors may use a simulation based on the classic Monty Hall problem or a site location decision scenario. In the site location scenario, an initial location for a distribution center is selected (among three possibilities) and the student must decide whether to change this decision based on new information that becomes available. Post workshop results showed that students found this activity to be an effective way of learning about the surprising decision a proper quantitative analysis provides. For faculty teaching probability concepts in operations and supply chain management courses at both the undergraduate and graduate levels, the workshop is useful in gaining student acceptance of the value in learning application of sometimes esoteric quantitative techniques. The exercise is also effective in encouraging critical thinking regarding initial impressions in probability-based decisions.

Keywords: probability, simulation, Bayes' Theorem, active learning, decision making.

1. Introduction

Demand for graduates with analytical skills continues to be robust, and resulting salaries are attractively high (Davenport and Patil 2012, Stanton and Stanton 2020). Understandably, demand for additional analytical content in operations and supply chain management courses has been surging (Schoenherr 2015). The media, legalized sports betting, and the growing popularity of other gaming industries (e.g., poker) have likely played important roles in helping spur student interest. In response to increasing industry and student demands, many universities and colleges now offer undergraduate and graduate programs focused extensively, or even exclusively, on analytics (Bennett 2020, Parry 2018).

Basic probability principles form important foundations in operations and supply chain analytics; they are integral for students studying quantitative material where elements of uncertainty are involved (demand planning, inventory management, facility location, project management, decision making, etc..). But of course, probability theory can be intimidating for students – especially those in required introductory business statistics and operations management courses (Wathen and Rhew 2019). Thus, it is challenging for instructors to teach these important foundations of analytics effectively, such that students obtain a good understanding of concepts and how to apply them in practice. The challenge is often compounded by the fact that people are generally not very good at accurately estimating probabilities (McGinty 2021), and that many important realities of probability are counterintuitive, making it unlikely that decision makers will scrutinize their initial judgements, or will accept that their judgements are wrong, even after an explanation (Camerer and Kunreuther 1989).

Recent pedagogical research demonstrates that use of various Excel simulation-based class exercises serve as innovative and effective ways to convey complex operations research concepts (Evans 2000, Weltman and Tokar 2019). Patterson *et al.* (2010) describe the Monty Hall Problem and provide an Excel simulation illustrating the optimal strategy and Law of Large Numbers convergence to expected probabilities. Our work adds important operations management applications, education on Bayes' Theorem, and teaching effectiveness results to this base. To this end, we present a simulation exercise, designed to be conducted in a classroom workshop, to address some of the challenges associated with teaching a complex yet critical probability concept; conditional probability and Bayesian updating. Application of Bayes' Theorem in business decision support is common and appropriate. For example, Grover (2013) provides a large variety of references and applications of the theorem for decision support in business, science, and engineering. Using an Excel application to immerse students in a classic, thought-provoking, and fun probability problem game, our workshop aims to provide a clear, hands-on learning experience with this difficult concept. Students realize that their intuitive probability estimates are not always accurate. Through our workshop exercise, students gain an appreciation for the importance of probability in making good business decisions. Our post-workshop survey results provide good support regarding its effectiveness.

The following sections of this paper describe the workshop in detail and present some evidence of its effectiveness as a learning tool. We then discuss our findings and the broader usefulness of the exercise.

2. Workshop Scenario: The Monty Hall Problem

Our workshop is based upon the well-known Monty Hall Problem (MHP), which was most notably displayed in the classic television game show, *Let's Make a Deal,* hosted by Monty Hall, hence the name of the problem. The show originally ran from 1963 until 1976, but was revived in 2009, now hosted by comedian Wayne Brady, introducing a whole new generation to the statistical dilemma. Schuller (2012) describes the MHP as follows:

> Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2 or stay with door No. 1?" Is it to your advantage to switch your choice?

In short, the central question for a contestant facing the MHP is whether to keep their originally selected door or switch to the other unopened door. To determine which course of action is best, he or she must properly assess the probabilities of the prize being behind either of the two unopened doors. Statistical analysis shows that there is, indeed, a correct strategy, however the answer is so highly counterintuitive that it is typically met with disbelief and has spawned heated debate, even among renowned mathematicians and scholars (Vazsonyi 1999; Dupont and Durham 2018).

The first scholarly manuscript regarding the problem appears in a letter written to *The American Statistician* (Selvin, 1975). It was addressed again (including the optimal strategy) a little more than a decade later (Nalebuff 1987). However, the MHP gained a great deal of academic attention beginning in September 1990 when famed *Parade Magazine* columnist Marilyn vos Savant wrote a response to a reader's question regarding the problem (vos Savant 1990a). vos Savant received over 10,000 letters regarding her correct answer to the problem, many from academics, and most of which argued (often quite rudely) that she was wrong (vos Savant 1990b). Her directive was that one should always switch doors if given the opportunity, as it increases the probability that one has selected the door containing the prize. The vast majority of people, at first glance, judge that it makes no difference whether one switches their choice, estimating the probability of either remaining unopened door containing the prize at 50%. In actuality, the chance of the player selecting the winning door increases from 33.3% to 66.6% if one makes the switch.[1] This reality is demonstrated quite easily using a decision tree or table, such as the one below in Table 1, adapted from Plous (1993).

Table 1: Possible outcomes and win probabilities in Monty Hall Problem

Prize is behind:	Participant's initial selection:	Host reveals:	Participant's action:	Participant's final selection:	Outcome:	Resulting win probability of action
Door 1	Door 1	Door 2 or 3	Switch	Door 2 or 3	Lose	2/3
	Door 2	Door 3	Switch	Door 1	Win	
	Door 3	Door 2	Switch	Door 1	Win	
	Door 1	Door 2 or 3	Keep	Door 1	Win	1/3
	Door 2	Door 3	Keep	Door 2	Lose	
	Door 3	Door 2	Keep	Door 3	Lose	
Door 2	Door 1	Door 3	Switch	Door 2	Win	2/3
	Door 2	Door 1 or 3	Switch	Door 1 or 3	Lose	
	Door 3	Door 1	Switch	Door 2	Win	
	Door 1	Door 3	Keep	Door 1	Lose	1/3
	Door 2	Door 1 or 3	Keep	Door 2	Win	
	Door 3	Door 1	Keep	Door 3	Lose	
Door 3	Door 1	Door 2	Switch	Door 3	Win	2/3
	Door 2	Door 1	Switch	Door 3	Win	
	Door 3	Door 1 or 2	Switch	Door 1 or 2	Lose	
	Door 1	Door 2	Keep	Door 1	Lose	1/3
	Door 2	Door 1	Keep	Door 2	Lose	
	Door 3	Door 1 or 2	Keep	Door 3	Win	

Nevertheless, the notion that one should always switch in this context remains difficult for many people to accept, as von Savant experienced. Other contexts where biased judgements have been observed include medical decisions. Eddy and Clanton (1982) document results from asking physicians to estimate the probability that a patient's tumor is malignant given: a) the chance of this type of tumor being malignant is 1%, and b) the tumor tests positive (malignant) with an instrument that correctly classifies 80% of malignant tumors and 90.4% of benign tumors. The correct probability of the patient's tumor being malignant is 7.8%, however 95 out of 100 physicians offered a judgment of between 70% and 80%. The difficulty in both this context and the MHP come down to the fact that decision makers typically fail to accurately update prior probabilities based on a condition. In our three-door scenario, the original probabilities for any door containing a prize are one-third. When an unchosen door that is known to not contain the prize is opened, people often fail to realize the game is no longer random and the probability for the remaining unchosen door to contain the prize changes with this event or condition. In these contexts, or any other in which a related judgement is

1. These probabilities are based on the following assumptions: 1) The host will always open a door after your initial selection, 2) The host never opens the door you initially selected, 3) The host never opens the door with the prize behind it, 4) The car is equally likely to be behind any door, and 5) If the player's initially selected door contains the prize, giving the host two choices of doors to open, the host chooses at random.